Spatio-temporal comparison of initial perturbation methods

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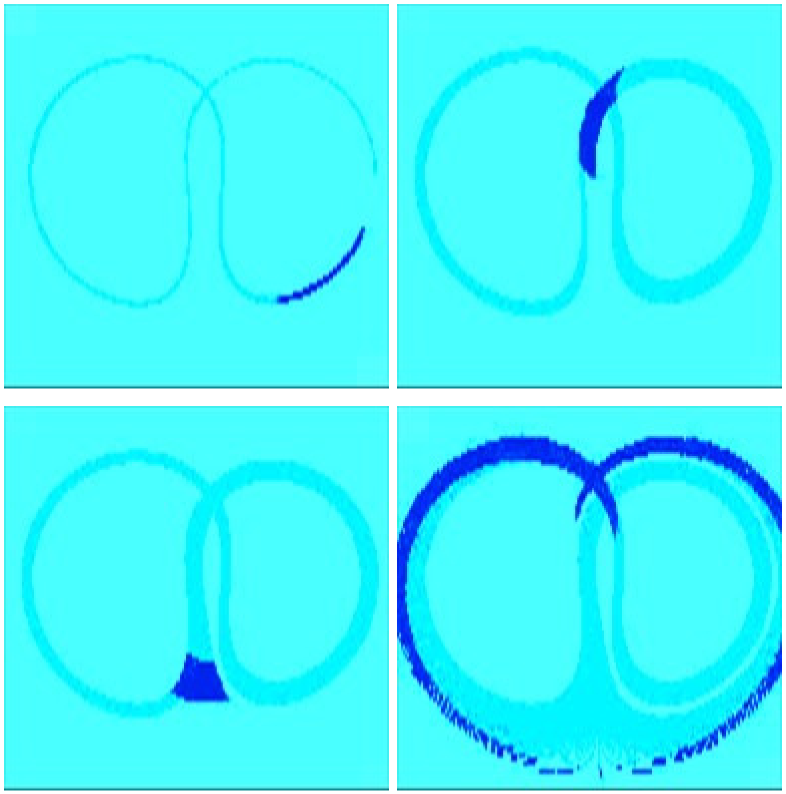
Outline

- Uncertainty in the initial conditions
- Perturbations growth in toy models
- Interface theory and MVL diagram
- Comparison of initial perturbations techniques in NWP models
- Summary and conclusions

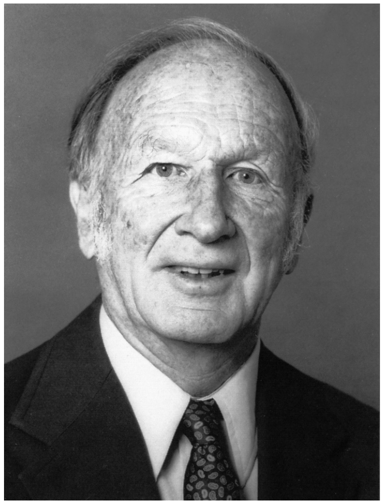
Chaos in the atmosphere: Lorenz model

Chaos was introduced by Edward Lorenz in 1963, when he used a simplified system to model the earth's weather on a computer.



$$
\begin{cases}
\dfrac{dx}{dt} = \delta(y - x) \\
\dfrac{dy}{dt} = \rho x - y - xz \\
\dfrac{dz}{dt} = xy - \beta z
\end{cases}
$$

$$\delta = 10, \rho = 28, \beta = \frac{8}{3}$$



"… one flap of a sea-gull's wing may forever change the future course of the weather" (Lorenz, 1963)

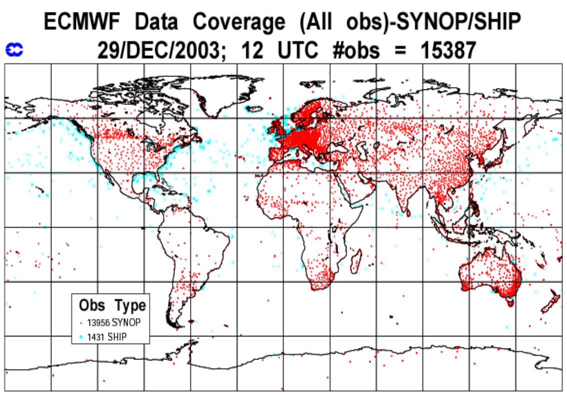
Small errors in the initial conditions become larger in time.

Sources of Uncertainty:

Uncertainty in the initial conditions:

- sampling error in measurements
- systematic error in measurements (e.g. instrumental biases)
- etc.

⇨ **Ensemble Prediction System**



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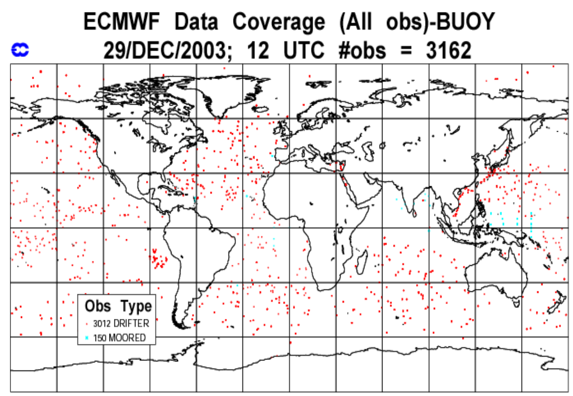
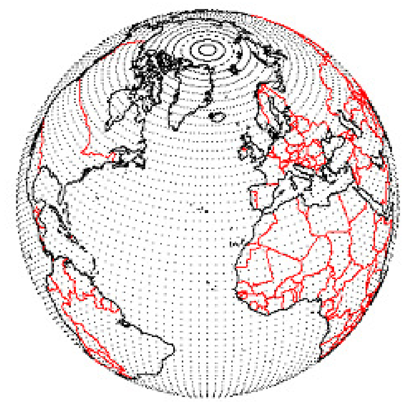


⇨



Uncertainty in the model:

- Physical uncertainty in model parameters
- Sampling uncertainty in statistical estimates of parameters
- Incomplete knowledge of external factors
- etc.

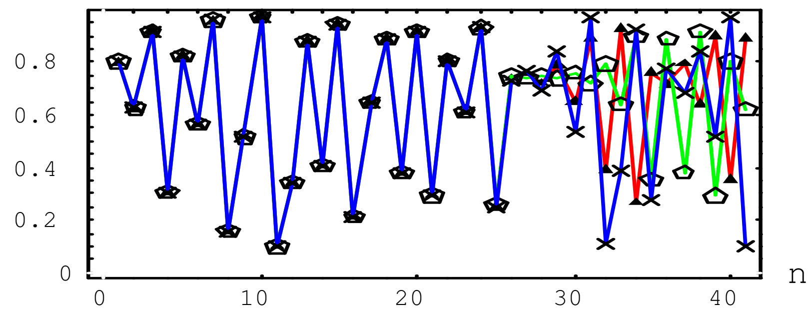
⇨ Multi-model

Toy models: Non linear dynamics & fractal geometry

Simple mathematical models allow us to understand the complex dynamic of nonlinear systems.

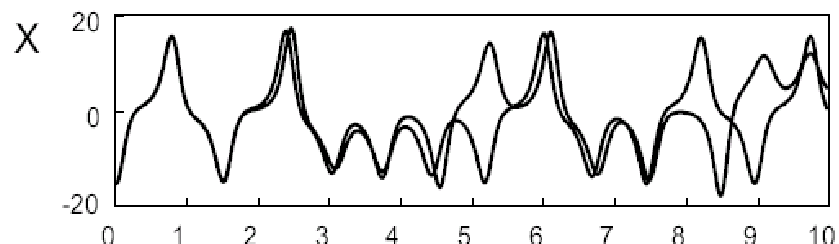
- Discrete case:
  Iterative systems
  Logistic maps

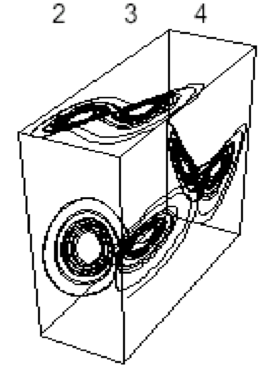
$$x_{n+1} = 4x_n(1 - x_n)$$



- Continuous case:
  Differential equations
  Lorenz systems

$$\begin{cases} \dfrac{dx}{dt} = \delta(y - x) \\ \dfrac{dy}{dt} = \rho x - y - xz \\ \dfrac{dz}{dt} = xy - \beta z \end{cases}$$

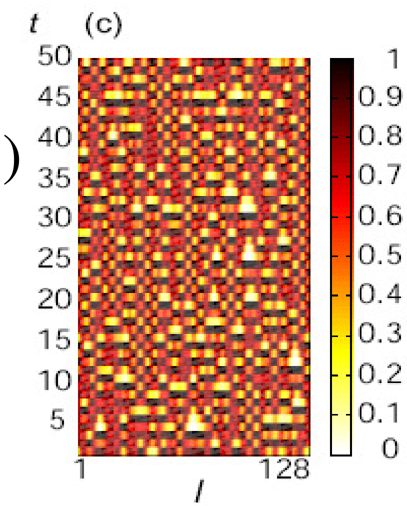




Toy models: Spatial coupling

Spatial points do not grow independently

Coupled chaotic maps:



$$u(x,t+1) = \varepsilon f(u(x+1,t)) + \varepsilon f(u(x-1,t)) + (1-2\varepsilon) f(u(x,t))$$

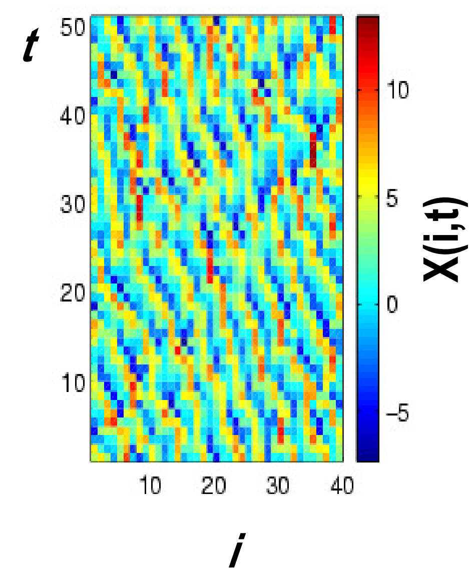
$$f(u) = 4u(1-u)$$ ***Logistic map***

$$u(x+L,t) = u(x,t), \;\; x = 1,\ldots,L$$ ***Boundary conditions***

$$\varepsilon = \frac{1}{3}$$ ***Homogeneous coupling***

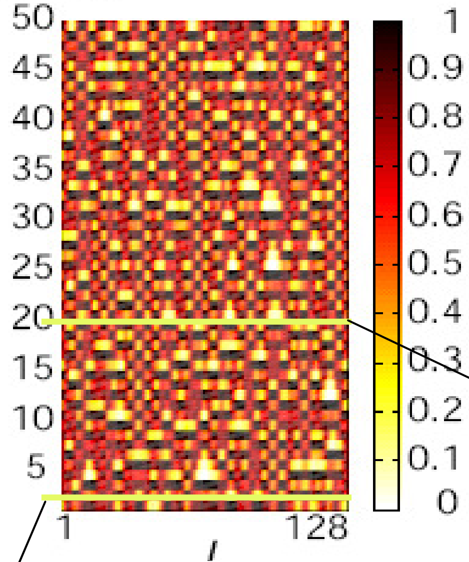
Lorenz-96 model:

$$\frac{dx_i}{dt} = -x_{i-1}(x_{i-2} - x_{i+1}) - x_i + F\,, \quad i = 1,\ldots,L$$



This model simulates the evolution of an atmospheric variable x in L grid points over a latitude

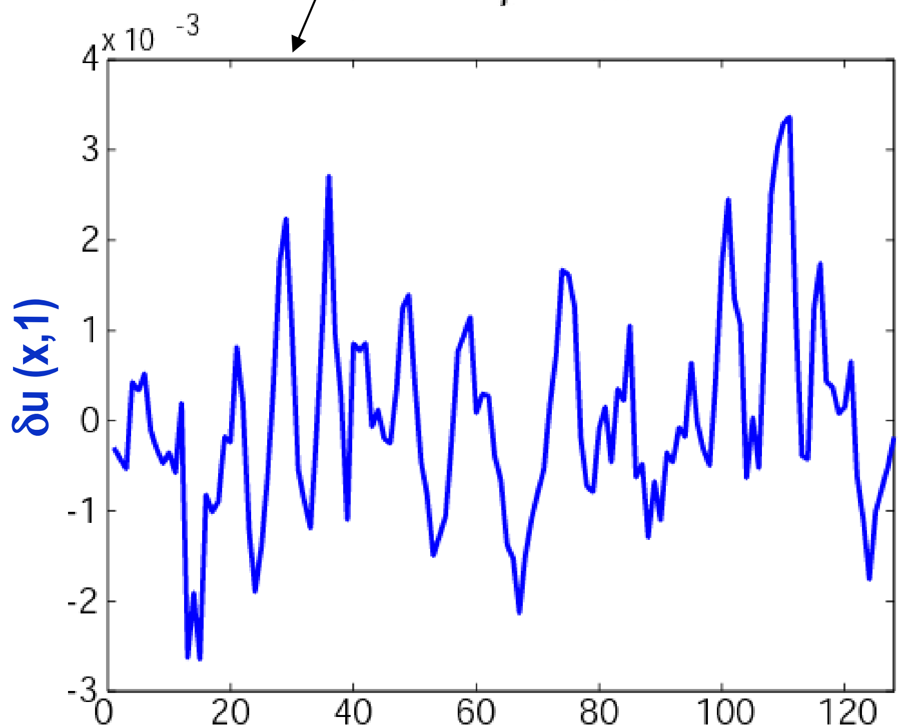
Spatio-temporal characterization of perturbations

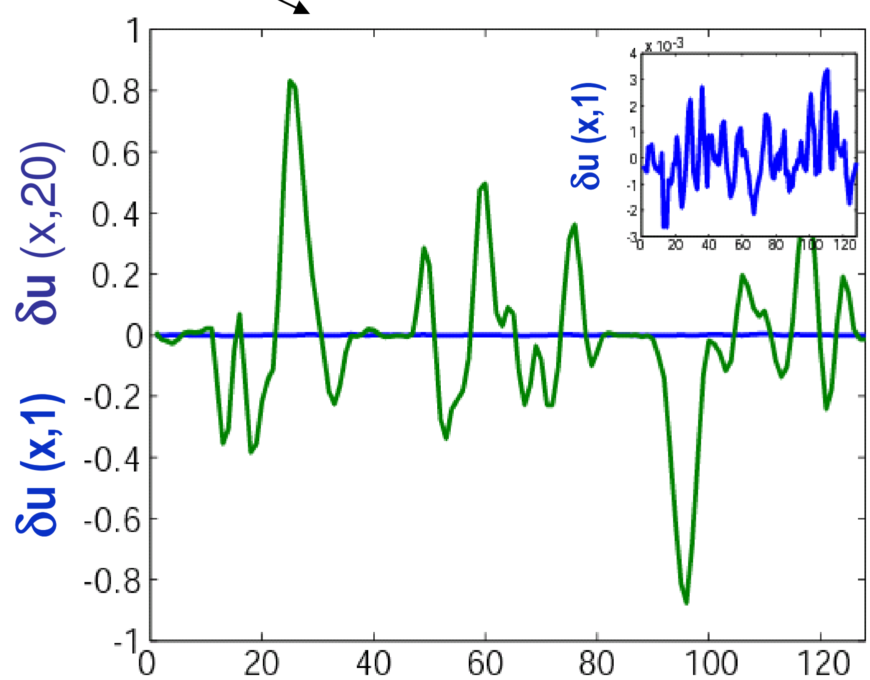


Perturbation **Perturbed trajectory** **Control trajectory**

$$\delta u(x,t) = u^{p}(x,t) - u(x,t)\,,$$

$$x = 1,\ldots,L$$



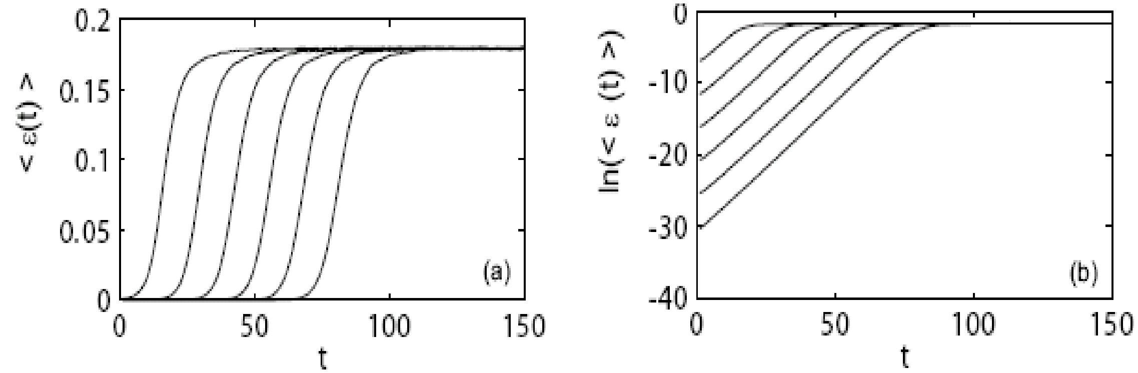


Temporal growth of the perturbations

- **Perturbations will exhibit a long-term average exponential growth with a characteristic rate:**

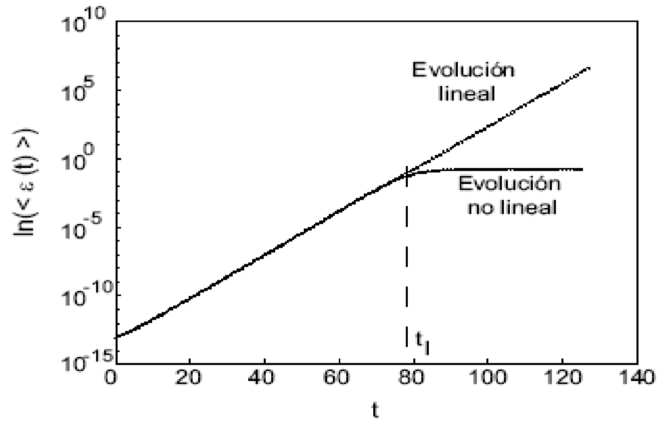
$$|\delta u(x,t)| \approx e^{\lambda_1 t} \qquad \text{where } \lambda_1 \text{ is the leading Lyapunov exponent}$$

$$\varepsilon(t) \equiv \prod_{x=1}^{L} |\delta u(x,t)|^{\frac{1}{L}}$$



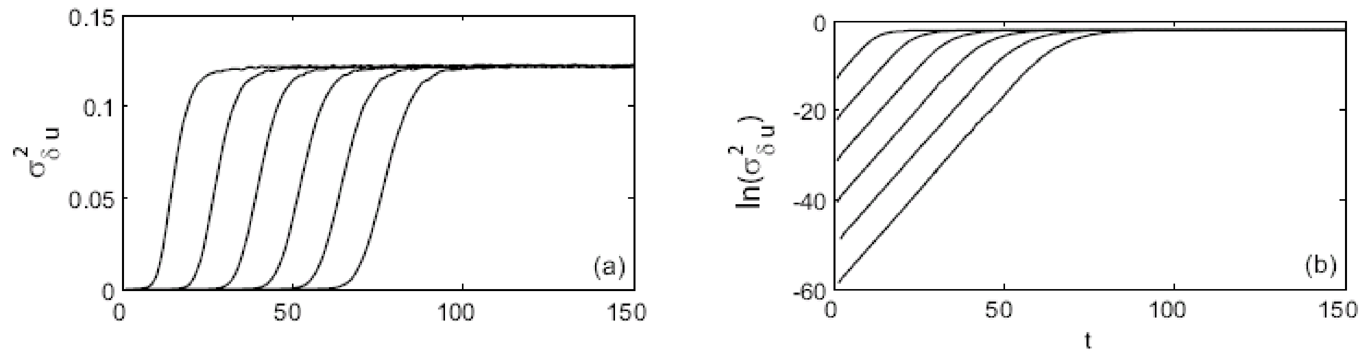
The lower the initial amplitude of the perturbations, the later they saturate.

In non-linear systems, perturbations become as large as the amplitude of the system.
When the system is linearized they do not saturate.

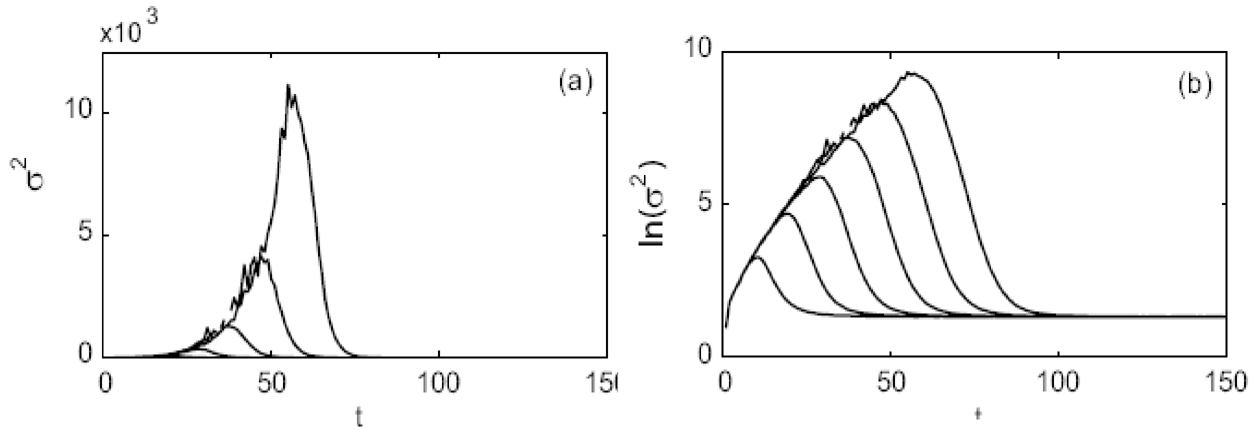


Spatial growth of the perturbations

$$\sigma_{\delta u}^2 = \frac{1}{L}\sum_{x=1}^{L}\left(\delta u(x,t) - \overline{\delta u(x,t)}_t\right)^2$$



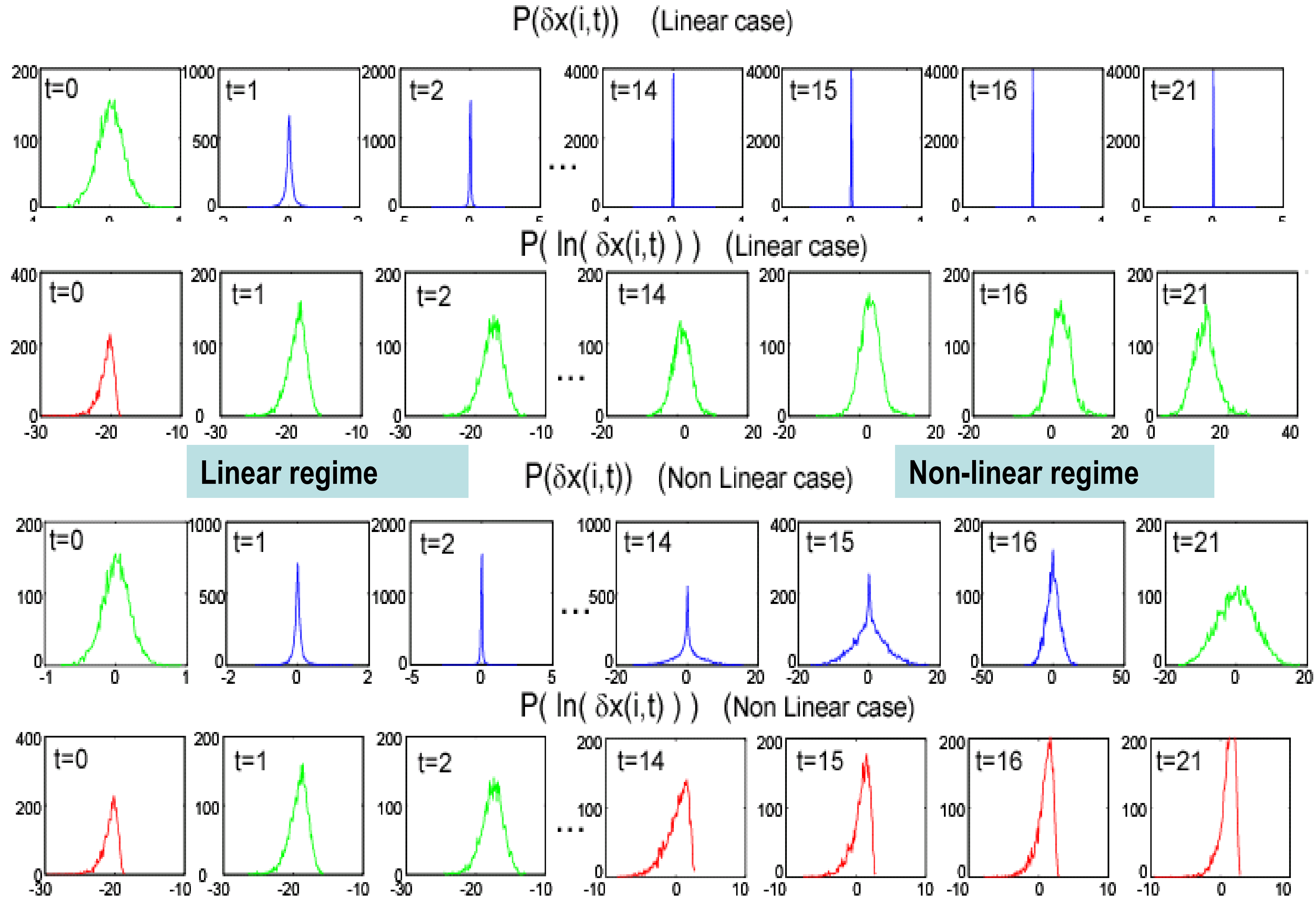
Dominated by the exponential growth. To avoid it:



$$\delta r(x,t) = \frac{\delta u(x,t)}{\varepsilon(t)}$$

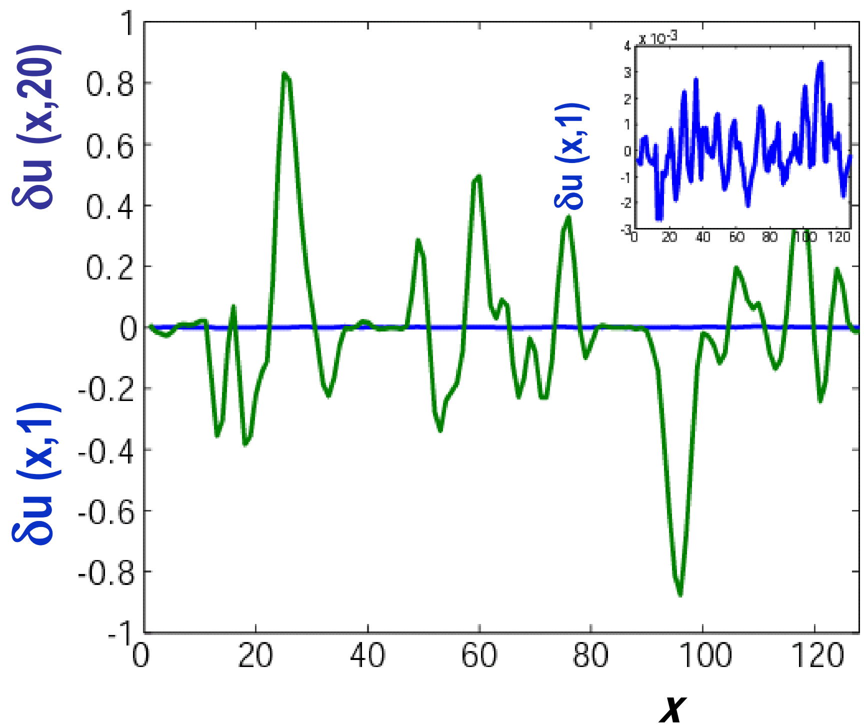
In the non linear regime, perturbations loose variance

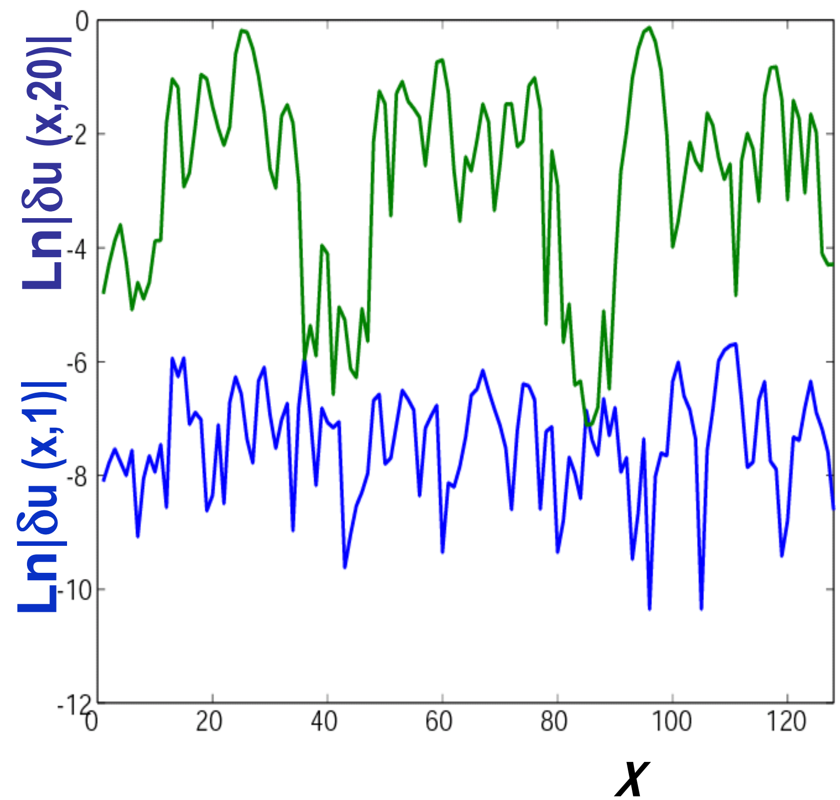
Localization



Logarithmic transformation

$$h(x,t) = \ln|\delta u(x,t)|,\ \ x = 1,\dots,L \quad \text{Hopf - Cole transformation}$$





There is a strong parallel between the logarithmic perturbations and the development of a rough interfase between two media.

Link between logarithmic perturbations and rough interfaces

- **Pikovsky & Kurths (1994) use this transformation to introduce roughening interfaces in the dynamics of perturbations of spatiotemporal chaos.**
- **Pikovsky & Politi (1998) apply it to study the dynamic localization of Lyapunov vectors in spacetime chaos.**
- **López et al. (2004) apply this correspondence to explain the scaling properties of growing noninfinitesimal perturbations in space-time chaos.**
- **Primo et al. (2006) introduce this link to characterize the dynamic scaling of bred vectors in spatially extended chaotic systems.**
- **Primo et al. (2007) show that the correspondence is also valid for the complex atmospheric circulation models used in weather forecasting.**

Interface growth

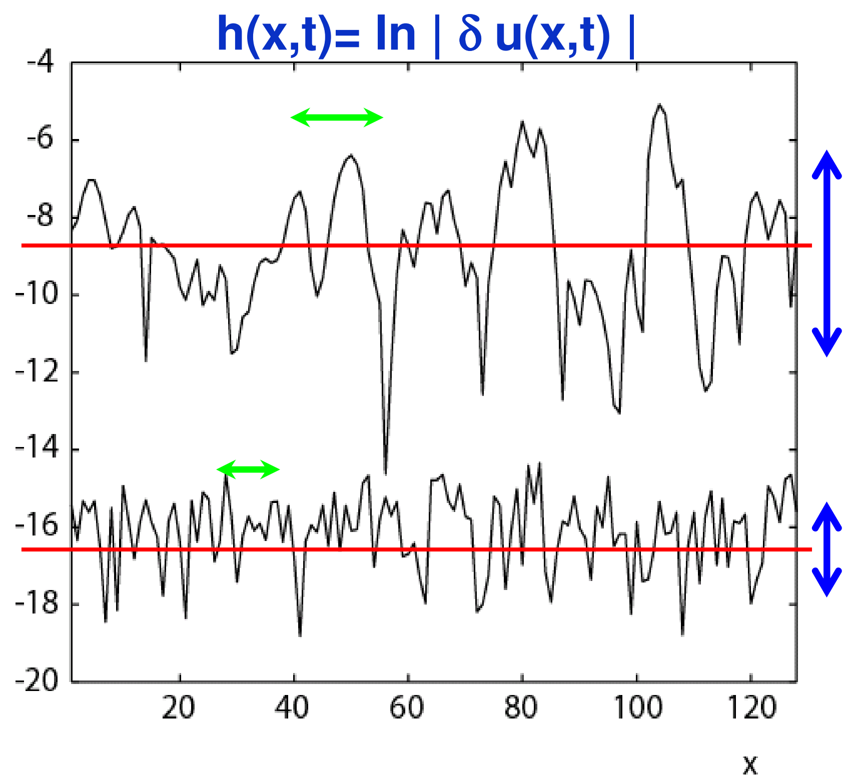
Mean height: $$\bar{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(x,t)$$

Width (standard deviation of the height):

$$w(t) = \sqrt{\frac{1}{L}\sum_{x=1}^{L}\left(h(x,t) - \bar{h}(t)\right)^2}$$

Correlation:

$$G(t,l) = \frac{1}{L}\sum_{x=1}^{L}\left(h(x+l,t) - \bar{h}(t)\right)^2$$



Width
(Vertical growth)

$$w(t) \approx t^{\beta}$$

$$w(t) \approx l(t)^{\alpha}$$

Correlation length
(horizontal growth) :

$$l(t) \approx t^{1/z}$$

Barabasi, A.-L. & Stanley, H. E. (1995): “Fractal Concepts in Surface Growth”. Cambridge University Press.

Mean-variance of logarithms diagram (MVL)

$$M(t) = \bar{h}(t) = \frac{1}{L}\sum_{x=1}^{L} h(x,t) = \frac{1}{L}\sum_{x=1}^{L} \ln|\delta(x,t)| = \ln\varepsilon(t) \approx \lambda_1 t + \ldots$$

$$V(t) = w^2(t) = \frac{1}{L}\sum_{x=1}^{L} (h(x,t) - \bar{h}(t))^2 \approx l(t)^{2\alpha}$$

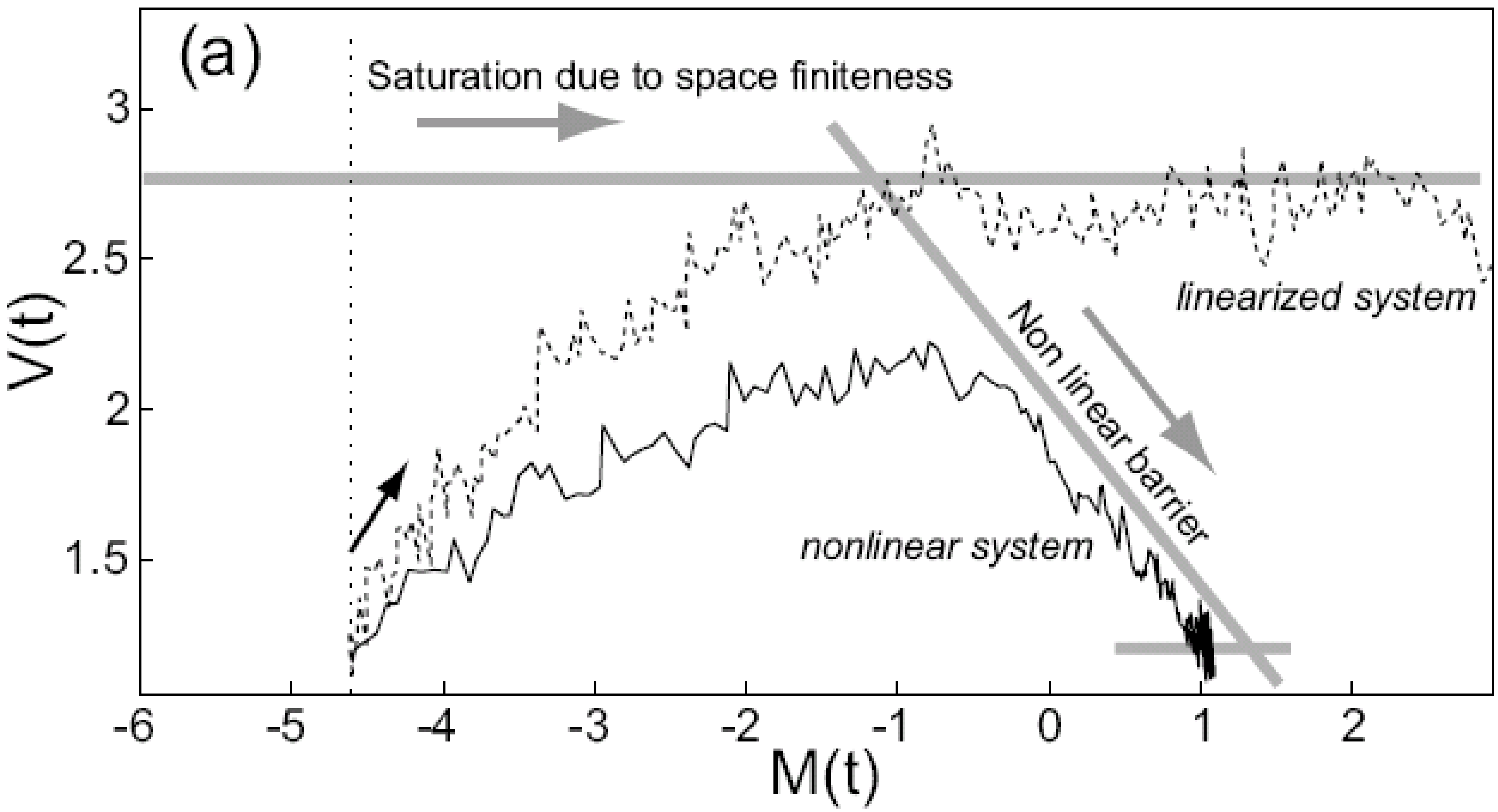


Figure: Gutierrez et al. (2008, Fig 2 (a))

MVL diagram

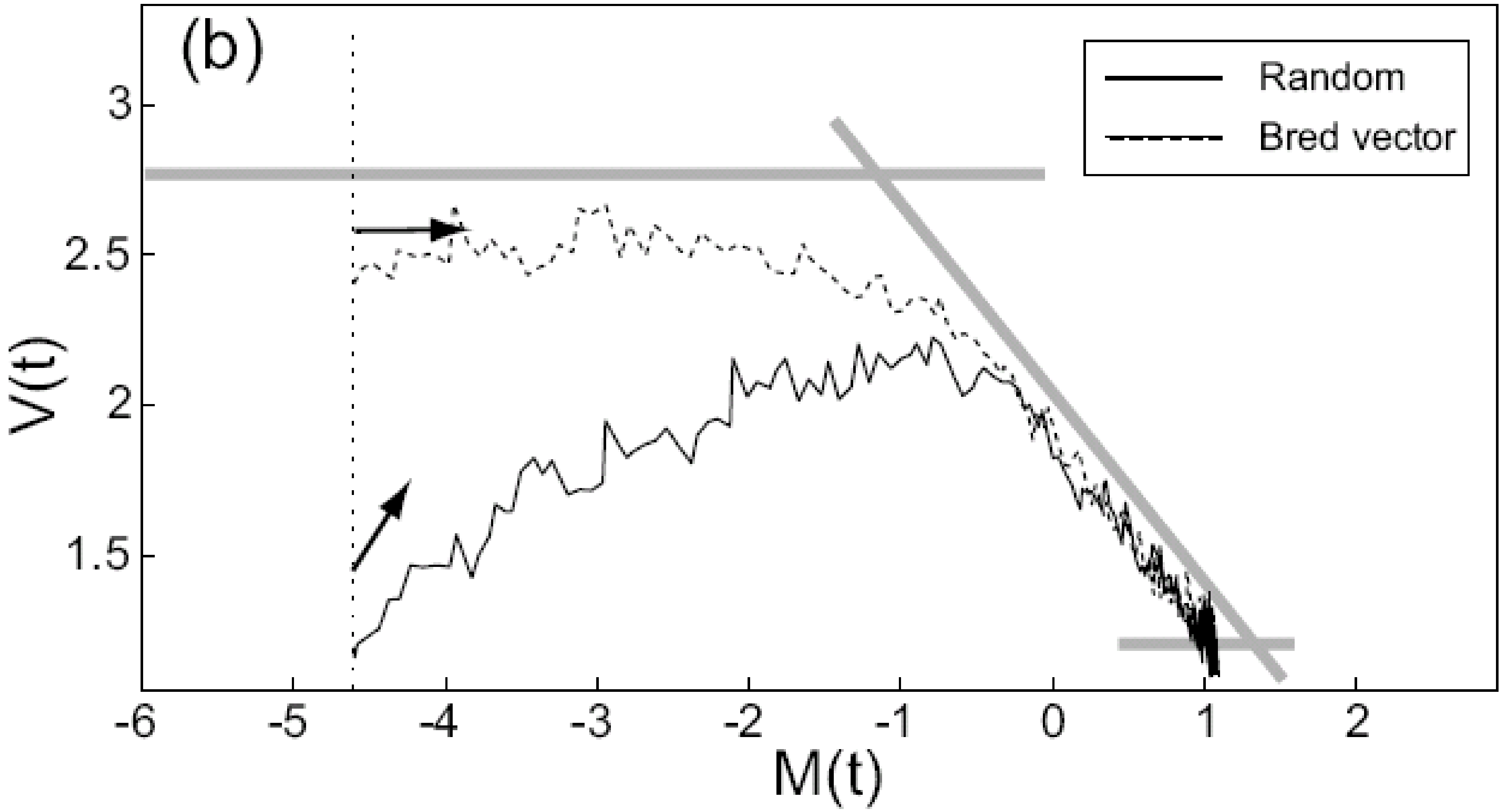
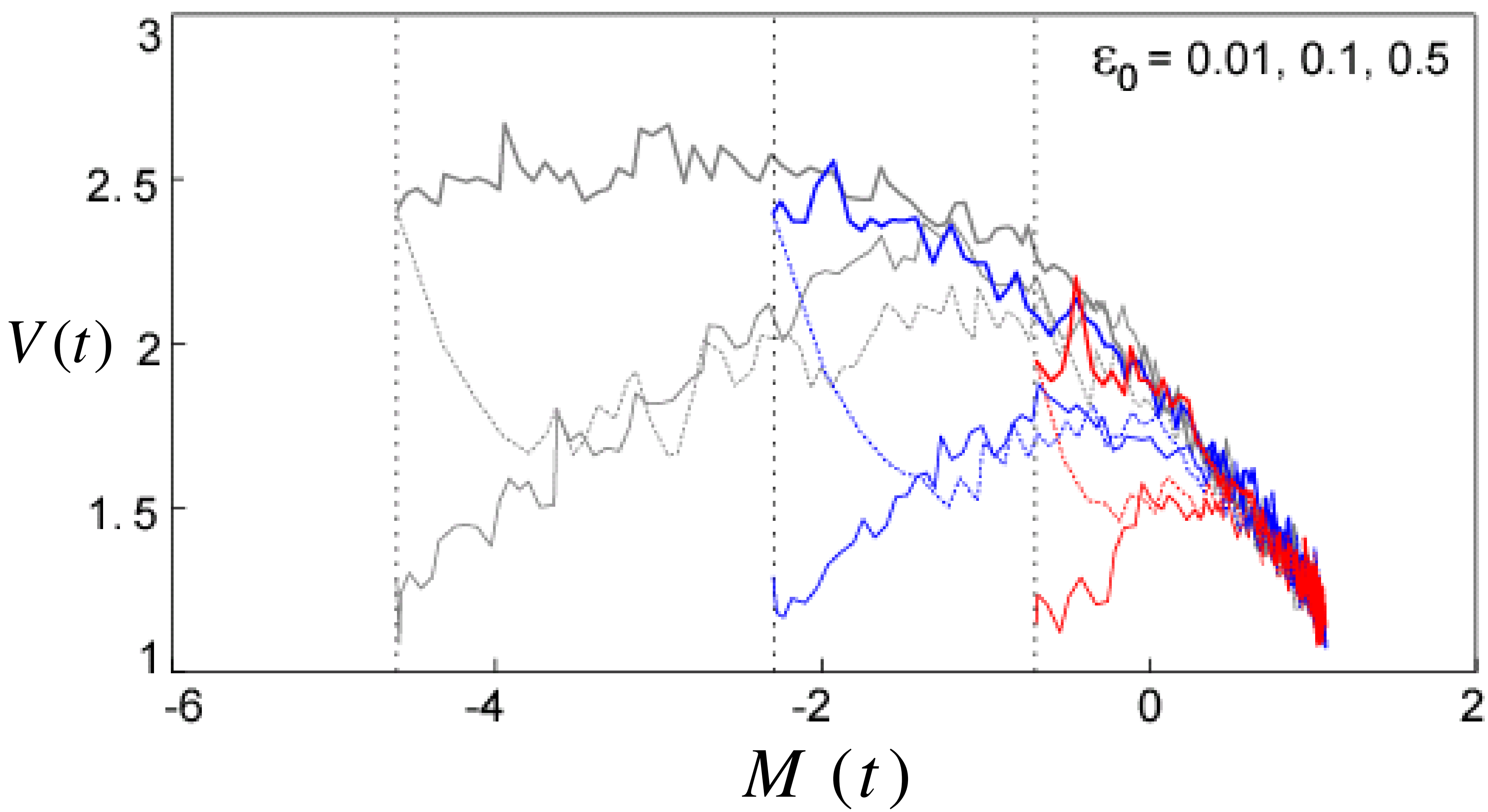


Figure: Gutierrez et al. (2008, Fig 2 (b))

MVL diagram



Numerical models

- **Operational forecast Centres represent uncertainty in the initial conditions by perturbing the best estimation of the initial state or analysis (ensemble forecasting).**

- **The nonlinear dynamics of numerical models makes perturbations grow exponentially in time and localize in space.**

- **There are different methods to perturb the initial conditions and the spatio-temporal growth of the perturbations differs from one technique to the other.**

- **The characterization of perturbations growth contributes to determine the predictability barrier of the system.**

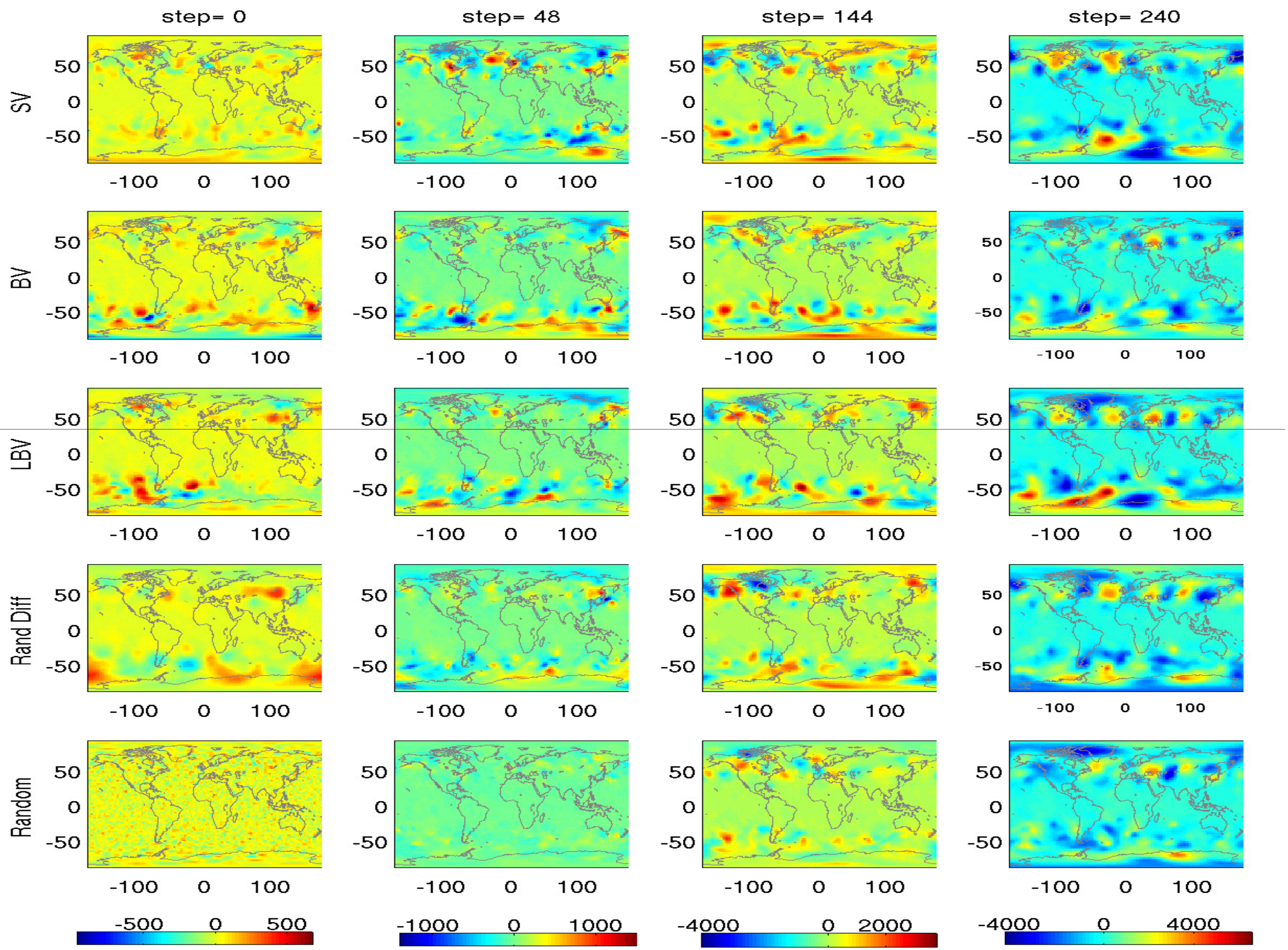
Experiment:

Spatio-temporal comparison of initial perturbation techniques:

- The operational singular vector system at ECMWF (SV), Lorenz 1965, Palmer et al (1993)
- Breeding vectors (BV) Toth and Kalnay (1993,1997)
- Breeding vectors rescaled by a geometrical mean (LBV) Primo et al (2005)
- Random field perturbation (RF) Magnusson et al (2008).
- Random perturbation technique

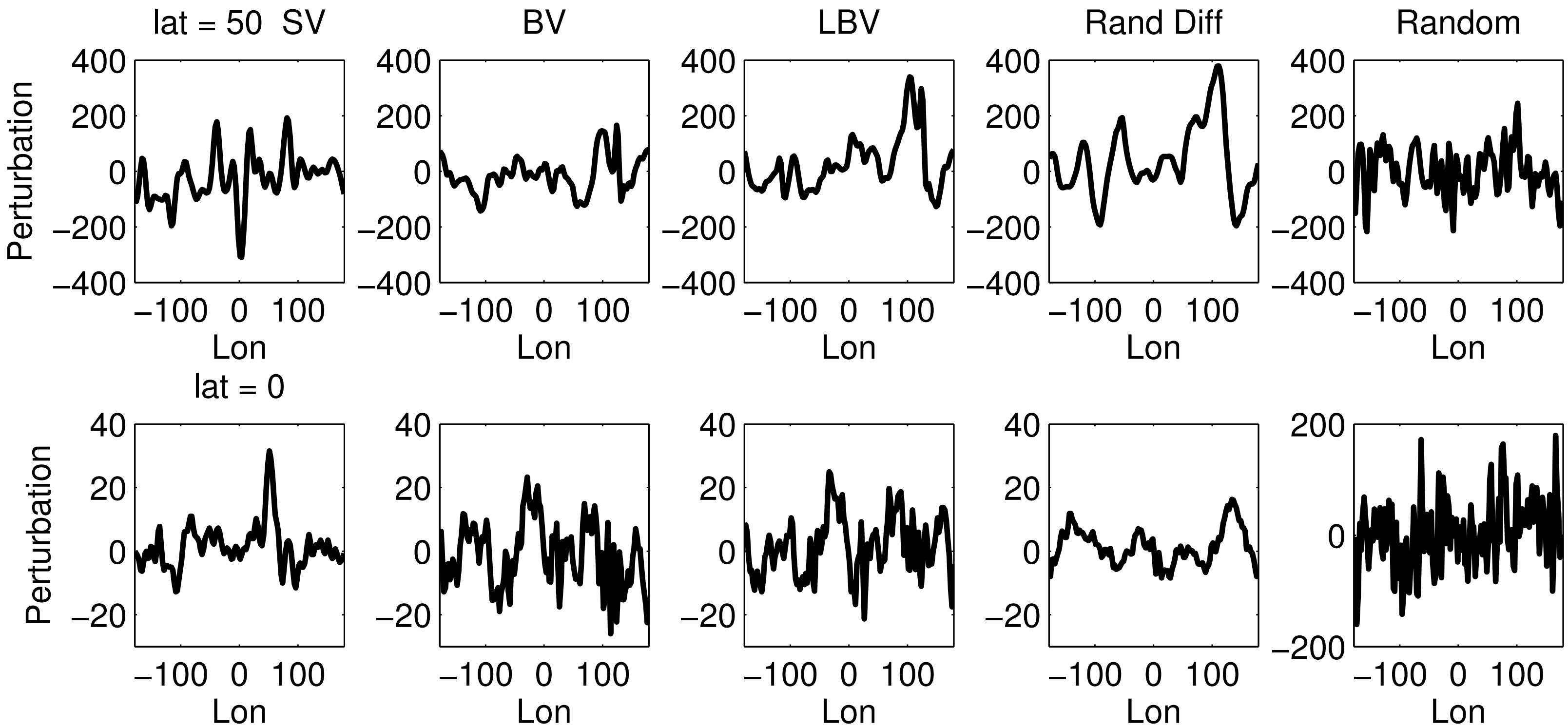
Experiment

- Data: Generated by Linus Magnusson (Stockholm University)
- Model: ECMWF Ensemble Prediction System.
- Period: 20050101-20050115
- Number of Members: 20
- Steps: from 0h to 240h by 12 h.
- Parameter: Geopotential
- Level: 500HPa
- Time: 00:00:00
- Different areas:
  North Hemisphere (90ºN, 30ºS, –180ºW, 180ºE),
  South Hemisphere (–30ºN, –90ºS, –180ºW, 180ºE),
  Europe (75ºN, 35ºS, –12.5ºW, 42.5ºE)
  Tropics (–30ºN, 30ºS, –180ºW, 180ºE).



step= 0
step= 48
step= 144
step= 240
SV
BV
LBV
Rand Diff
Random
50
0
-50
-100
0
100
-500
0
500
-1000
0
1000
-4000
0
2000
-4000
0
4000

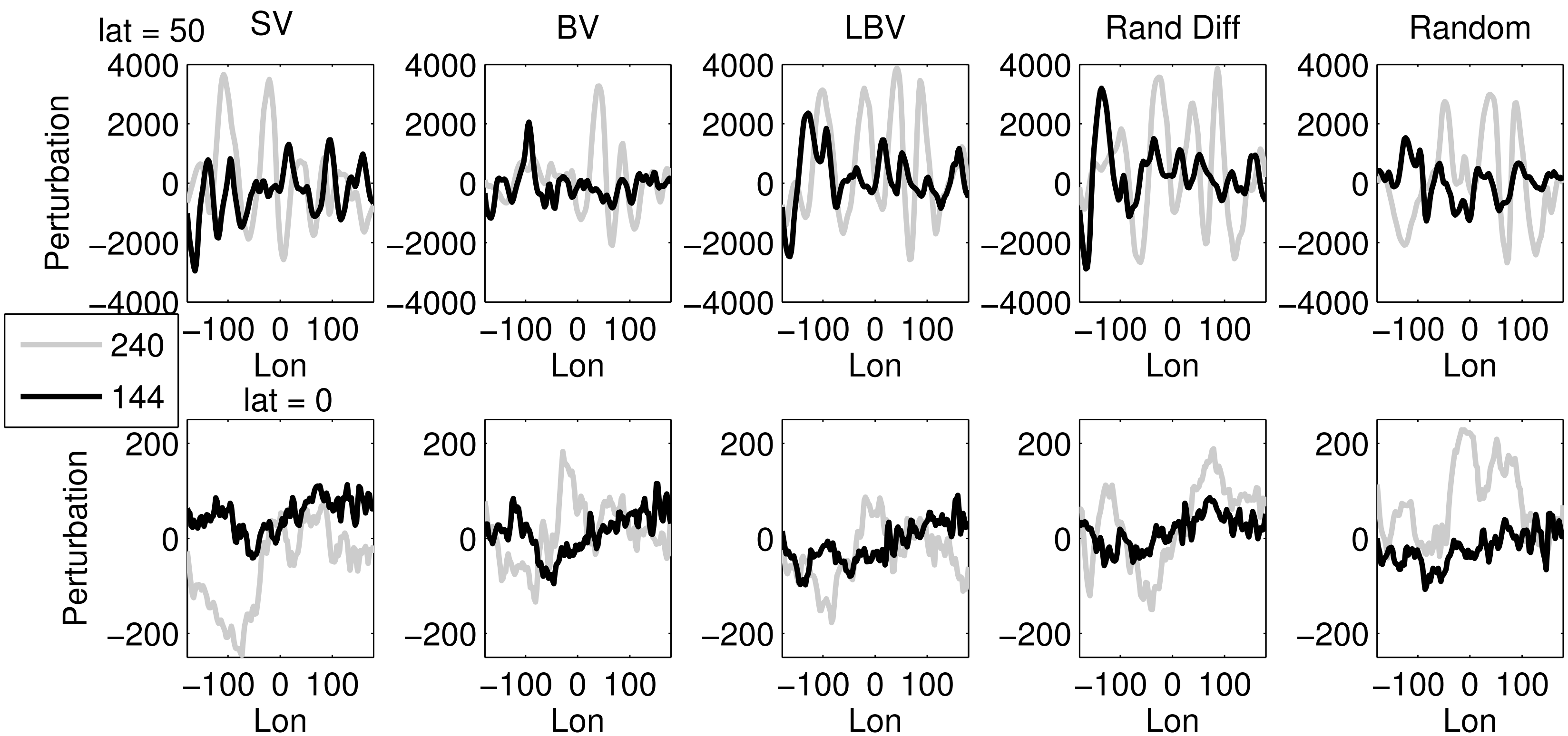
Initial perturbations



Perturbations are dominated by the extratropics, but the tropics also show spatial structure

Perturbations evolution

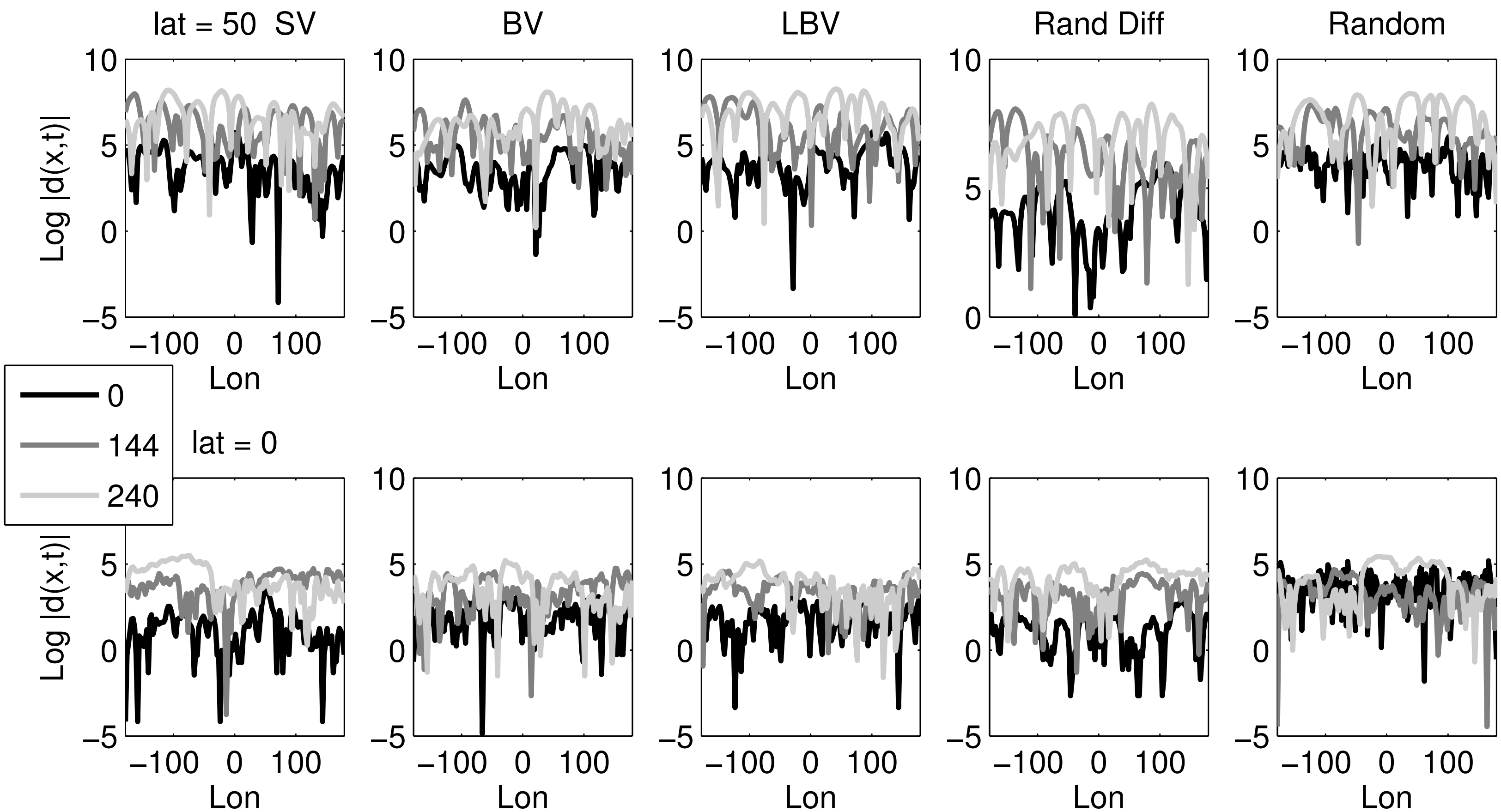
Perturbations after 6 (black) and 10 (grey) days



Spatial structures become bigger and tend to a similar pattern

Logarithmic perturbations evolution

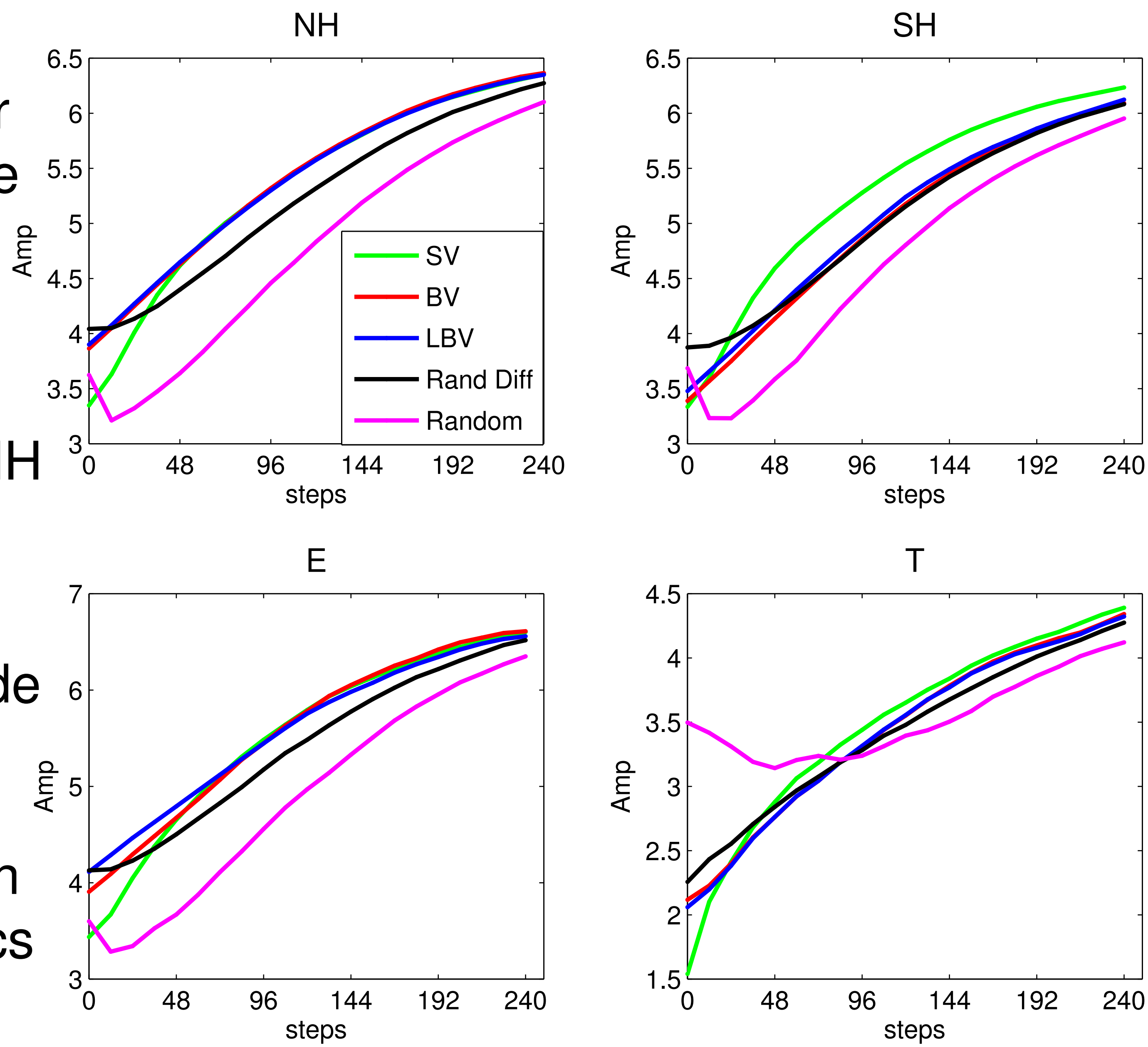
$$h(x,t) = \ln|\delta u(x,t)|, \quad x = 1,\ldots,L \quad \text{Hopf - Cole transformation}$$



Amplitude Growth

SV has smaller initial amplitude

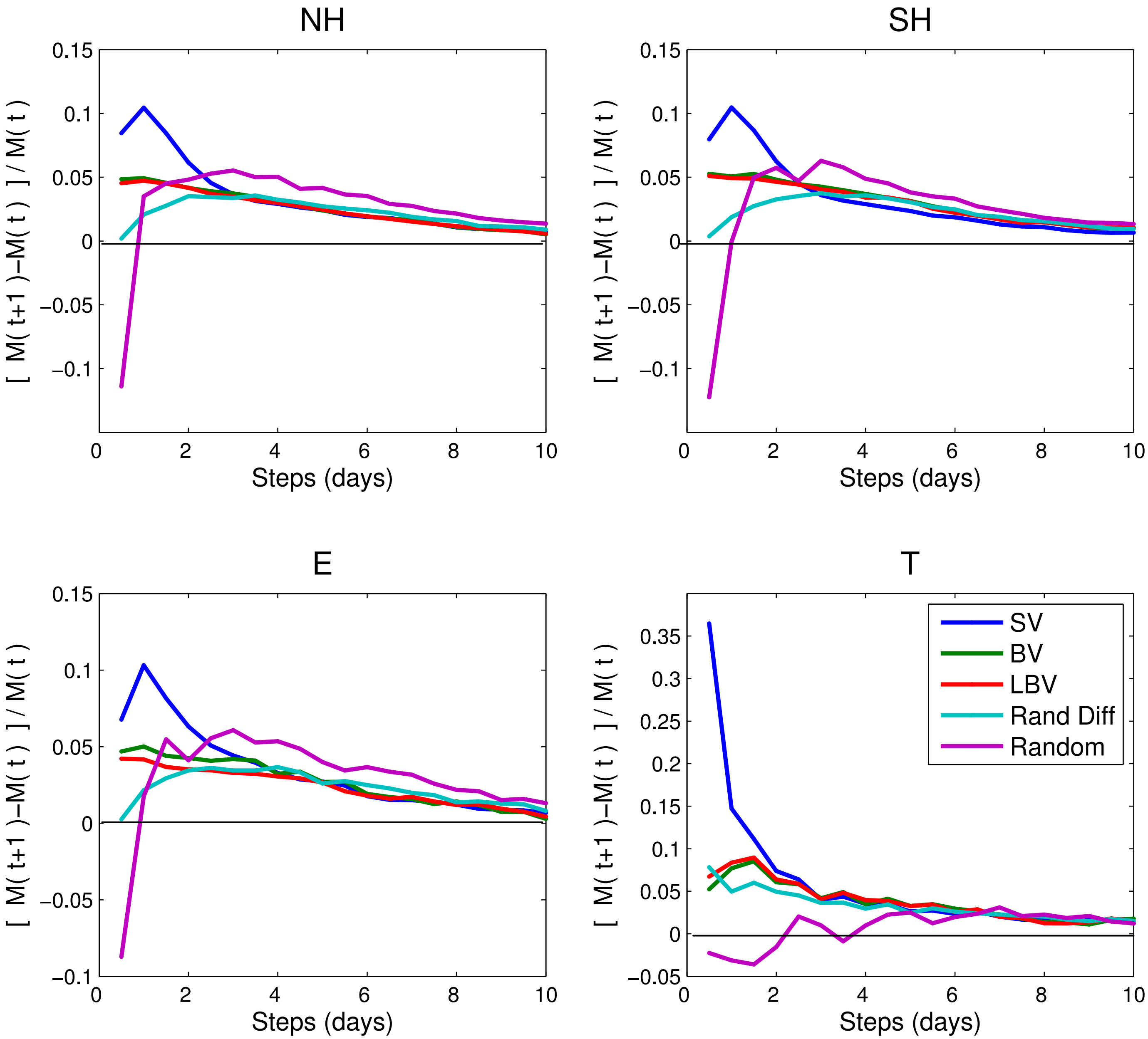
After two days SV and BVs amplitude is similar in the NH



Initial amplitude and growth is smaller in the Tropics than in the extratropics

Amplitude Growth rate:

$$Rate(t) = \frac{M(t+1) - M(t)}{M(t)}$$



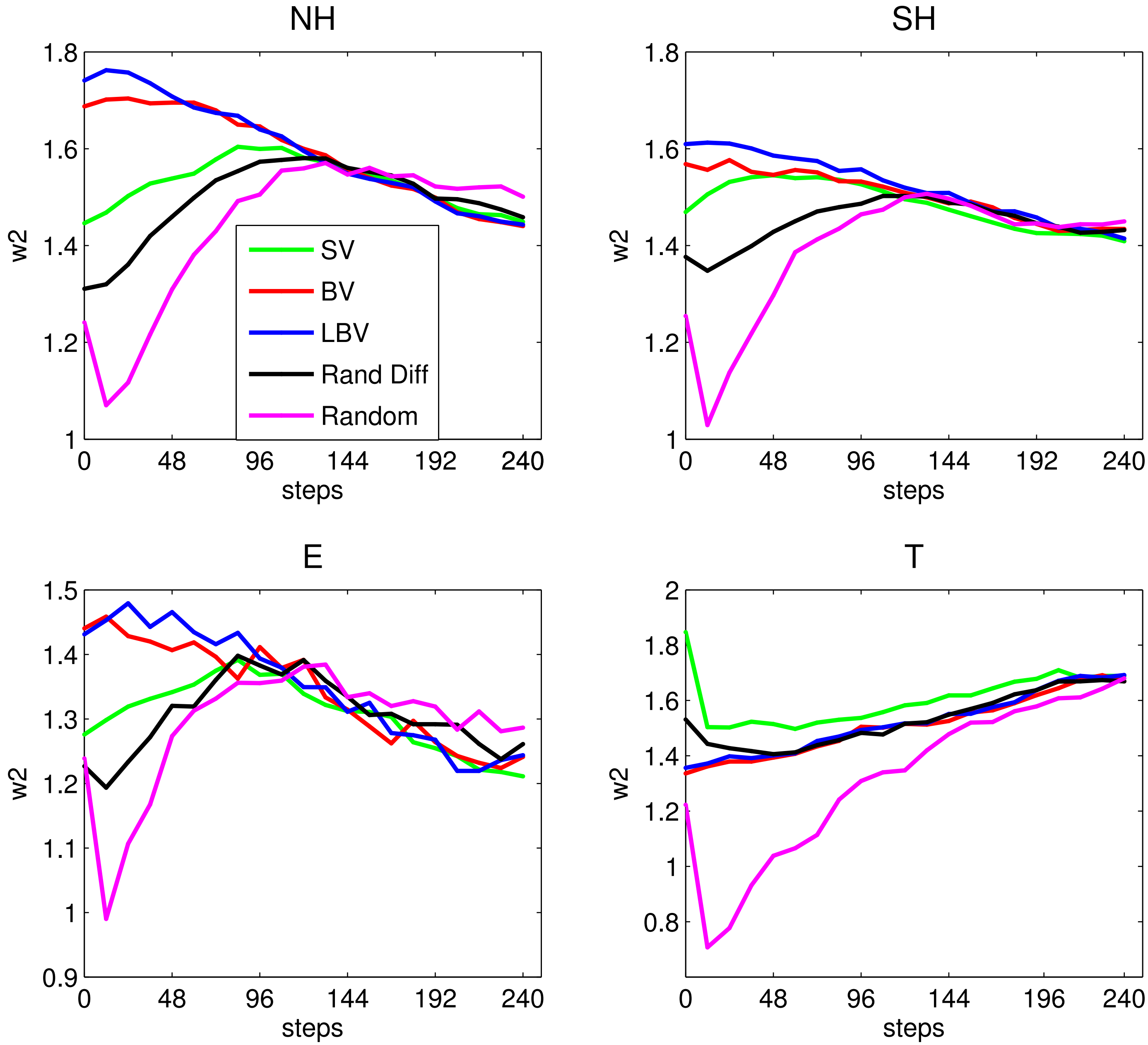
SV has fastest initial growth, but after around three days growth rate converges

Variance growth

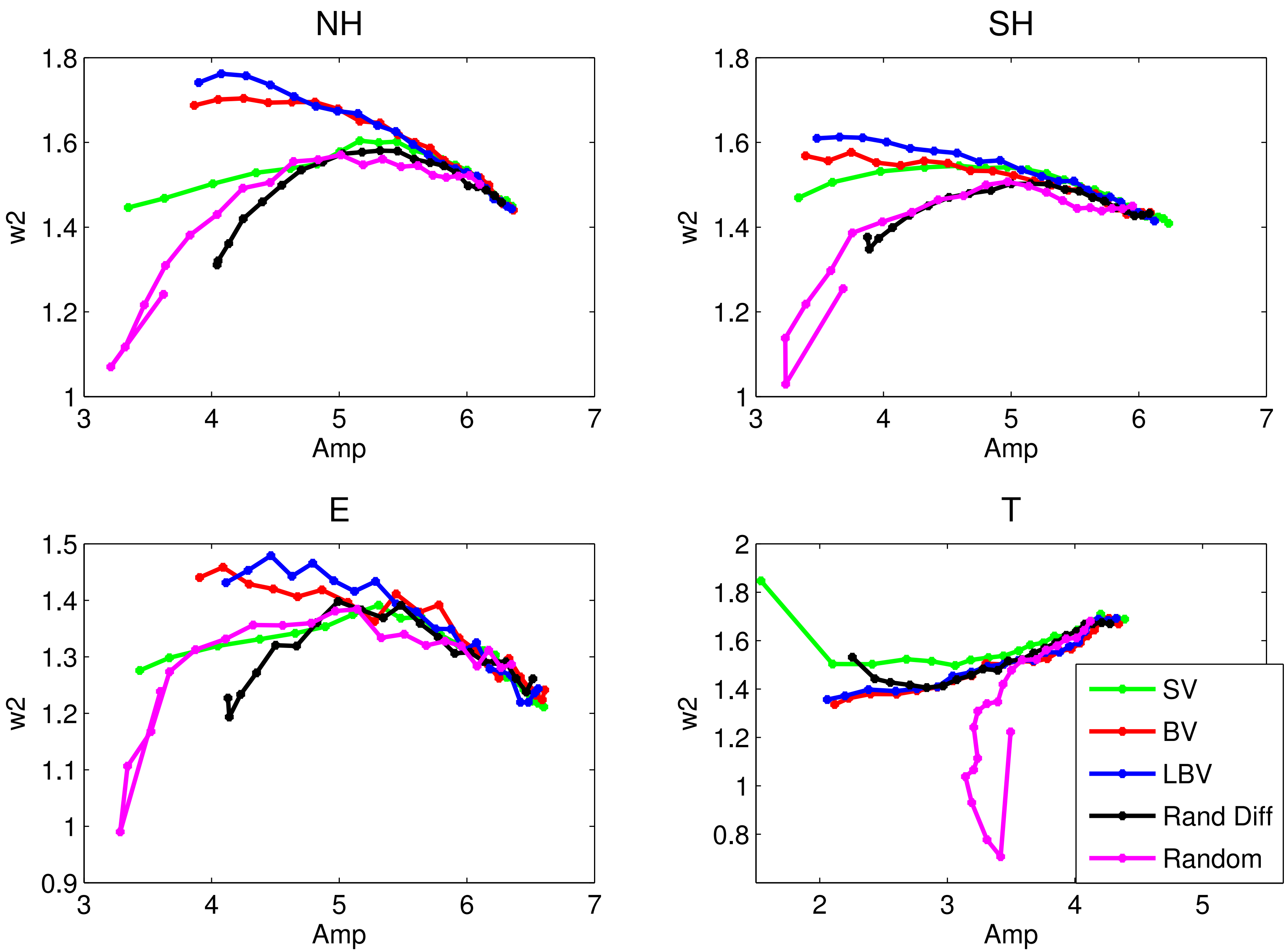
BVs provide perturbations with the highest correlation in the extratropics

Random perturbations start with lower correlation, but after around five days all techniques converge

Correlation in the Tropics seems not to be saturated after 10 days



MVL Diagram



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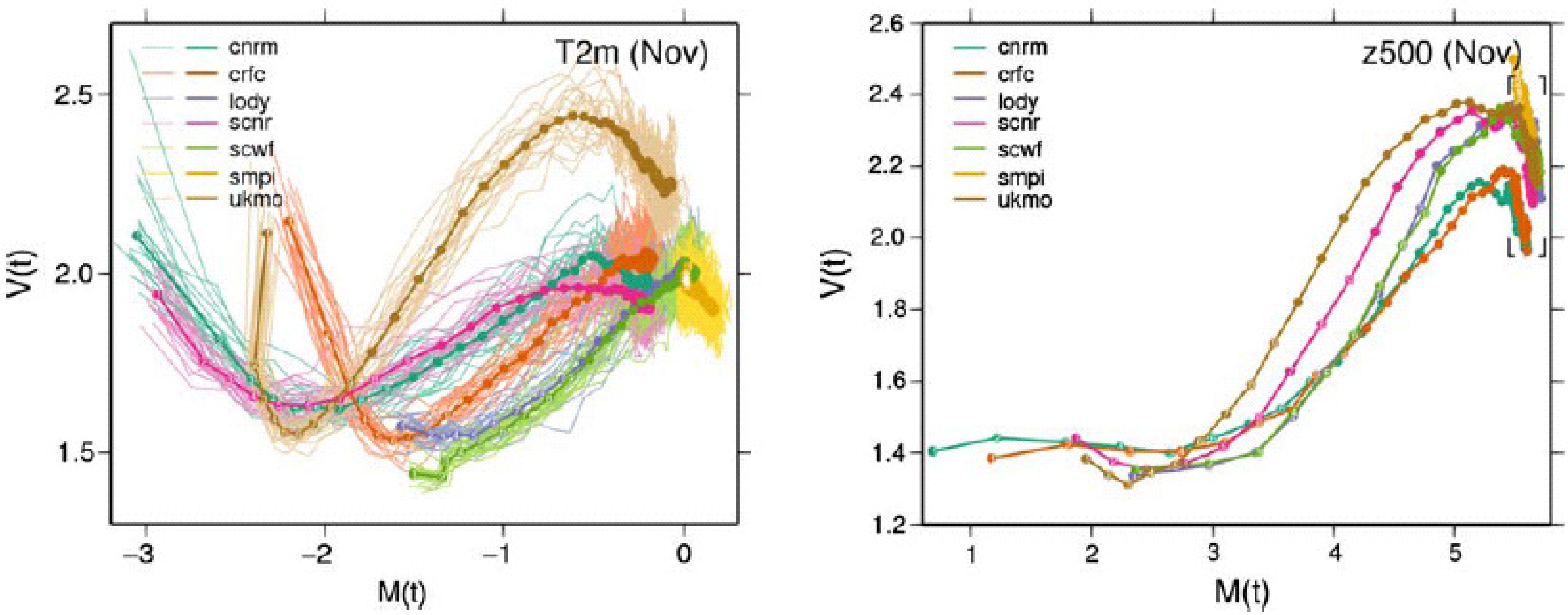
Seasonal forecasts: DEMETER

- The DEMETER project is a multi-model seasonal ensemble with hindcasts for seven models covering a common 22-year period (1980-2001).



Fernández et al. (2009, figs 2, 3)

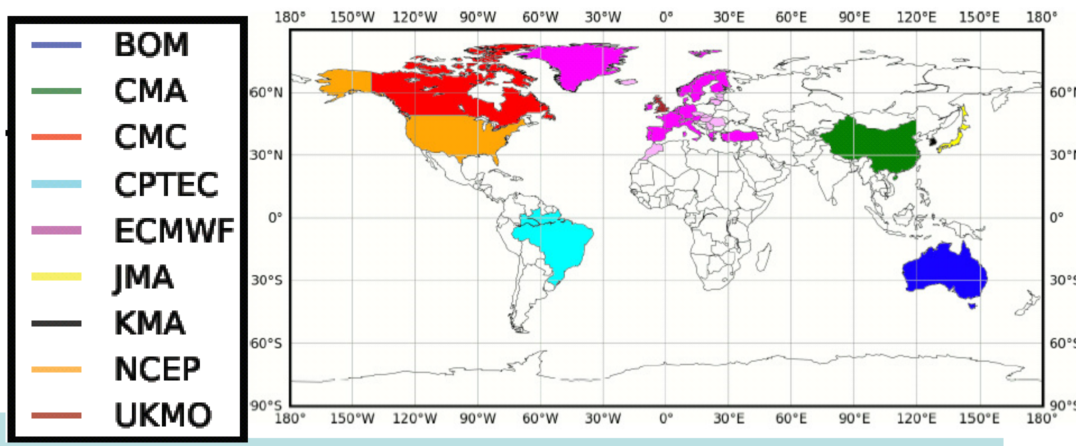
Medium-Range forecasts: TIGGE

TIGGE project provides an archive of medium-range ensemble forecasts from a range of different operational models in a consistent format.

Centre	Location	Horiz. res.	Levels	Members	Perturbations
BOM	Australia	T119	19	$1+32$	SV (initial)
CMA	China	T213	31	$1+14$	BV
CMC	Canada	0.9°	58	20	EnKF (since 2005)
CPTEC	Brazil	T126	28	$1+14$	EOF
ECMWF	Europe	T399	62	$1+50$	SV (initial+final) + stoch.
JMA	Japan	T319	60	$1+50$	SV (since Nov 2007)
KMA	South Korea	T213	40	$1+16$	BV
NCEP	USA	T126	28	$1+20$	ETR + stoch.
UKMO	UK	$1.25^\circ \times 0.833^\circ$	38	$1+23$	ETKF + IAU + stoch.

- **BV** Bred Vectors
- **EOF** Empirical Orthogonal Functions
- **EnKF** Ensemble Kalman Filter
- **ETKF** Ensemble Transform Kalman Filter
- **ETR** Ensemble Transform with Rescaling
- **IAU** Incremental Analysis Update
- **SV** Singular Vectors (initial- and/or final-time)
- **stoch.** Stochastic parameterisation

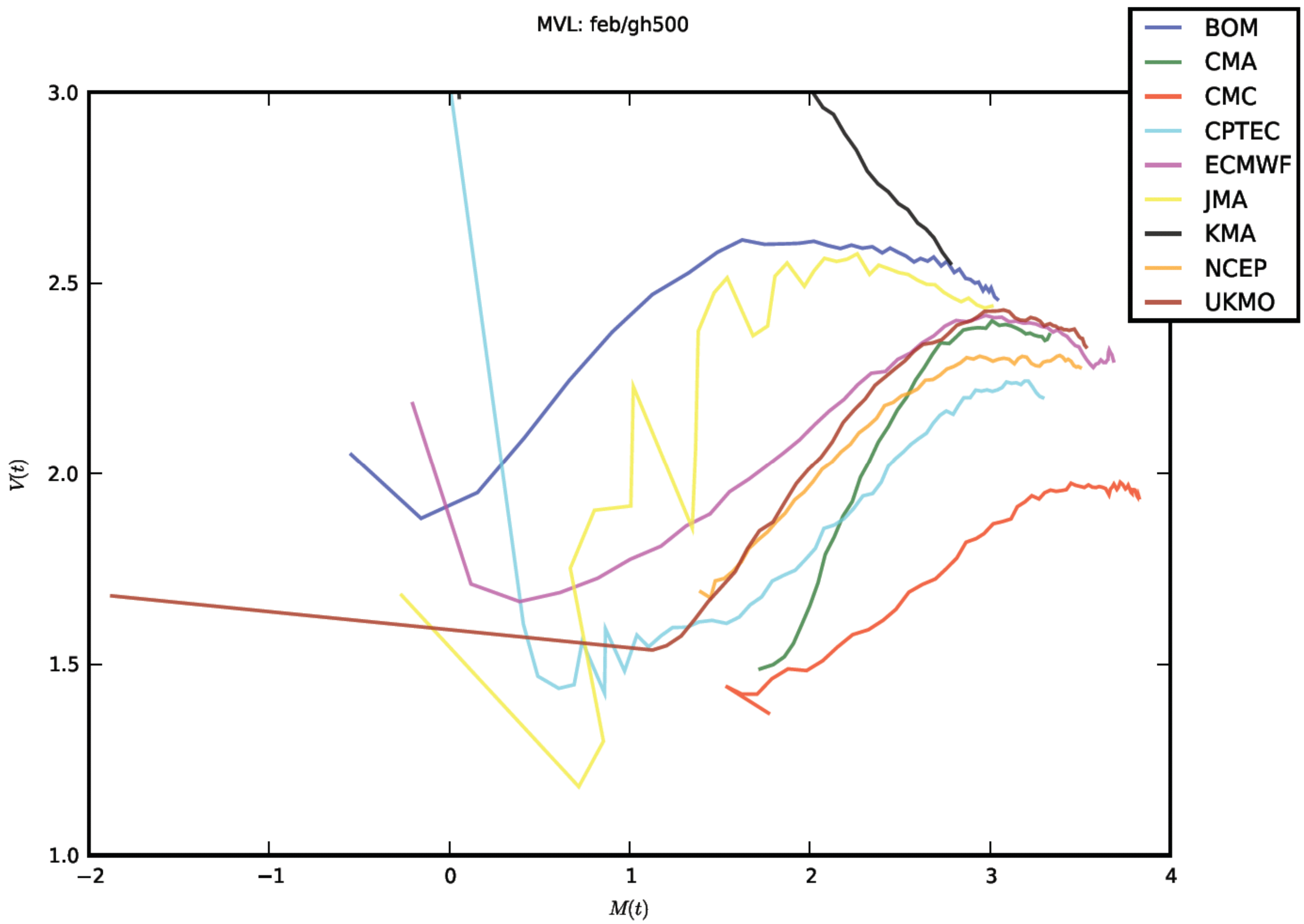
The regions of the globe over which perturbations are optimised differ from model to model



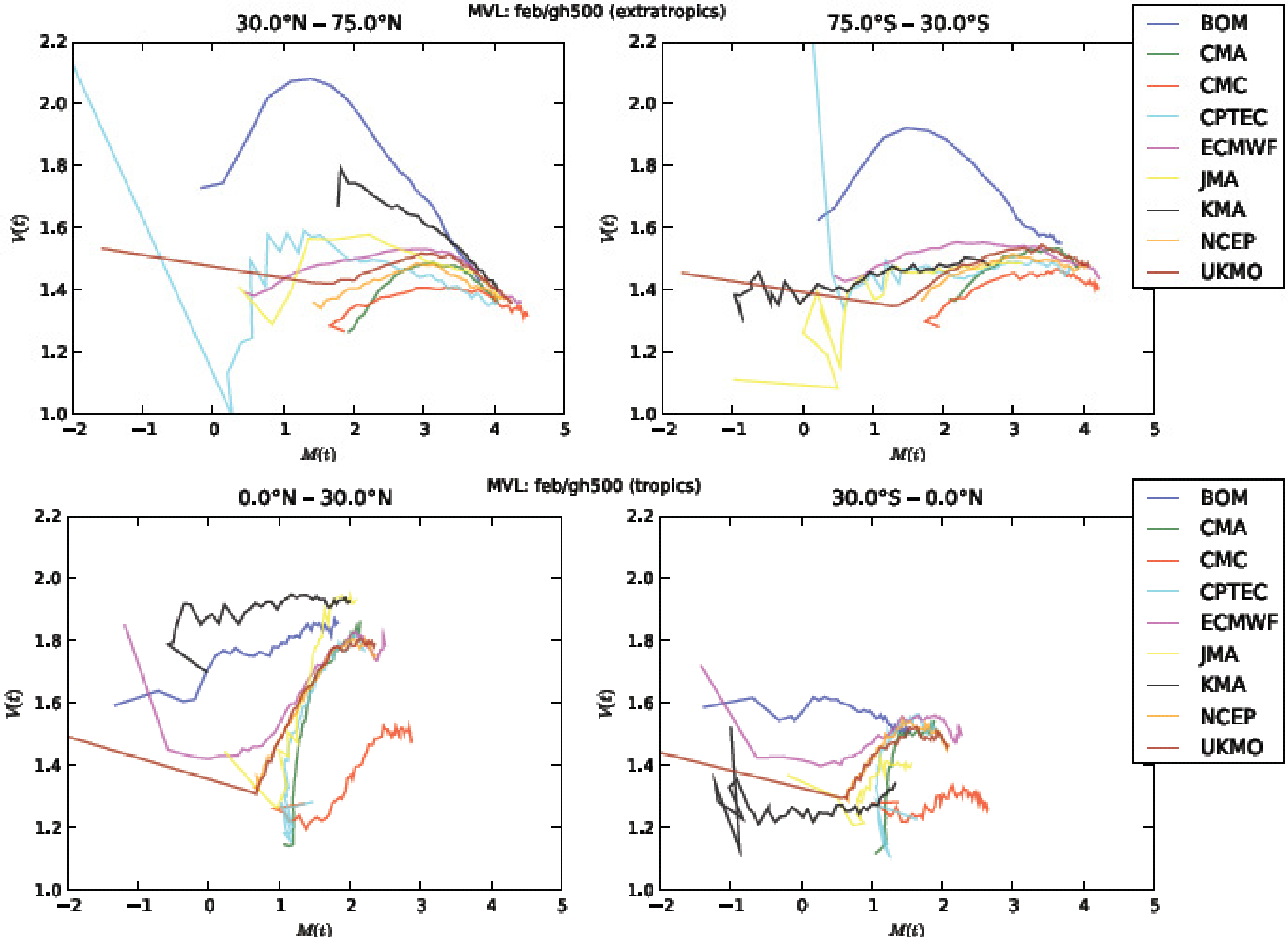
MSc Student: Zak Kipling

Project: Error growth in medium-range forecasting models

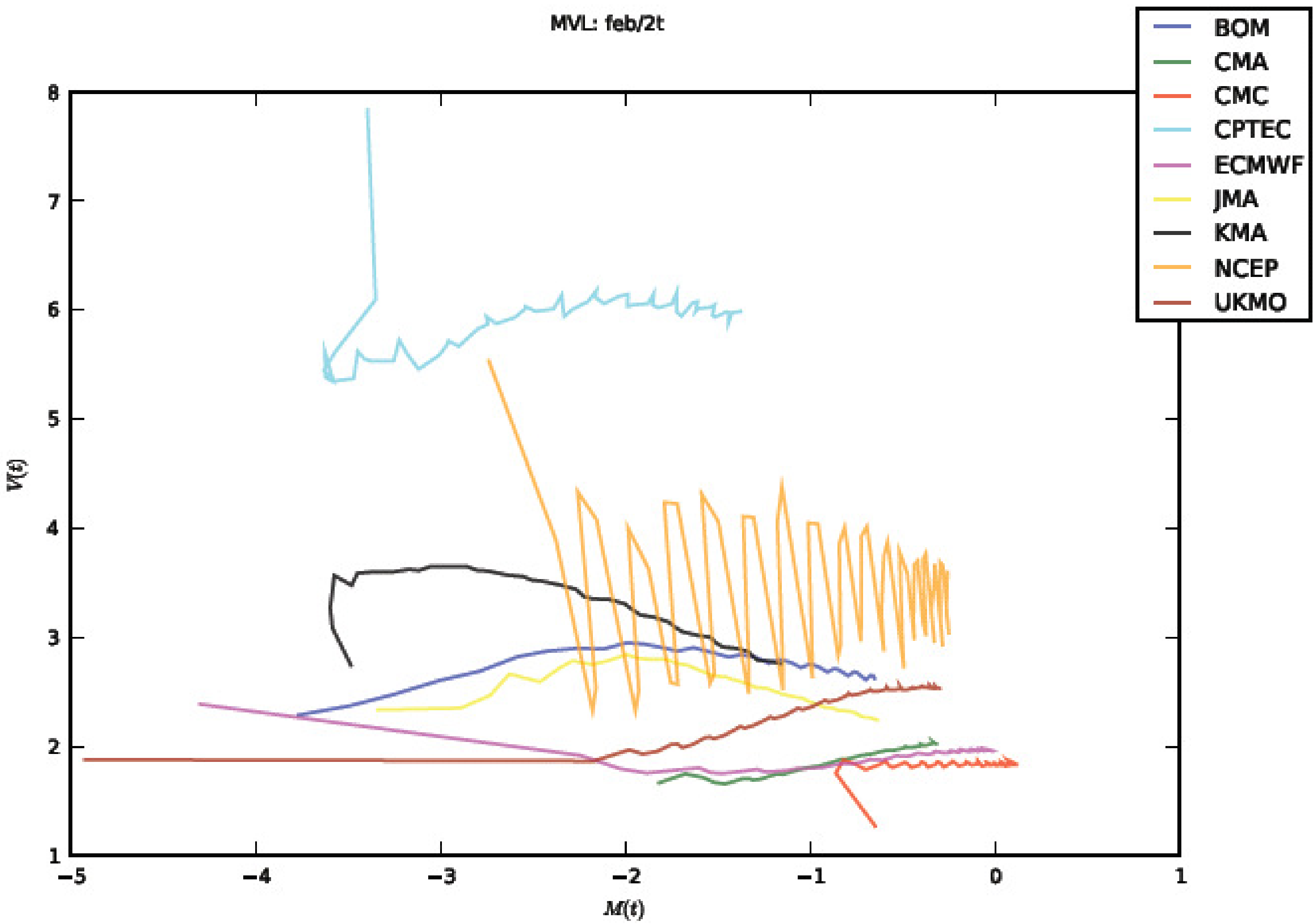
Medium-Range forecasts: TIGGE



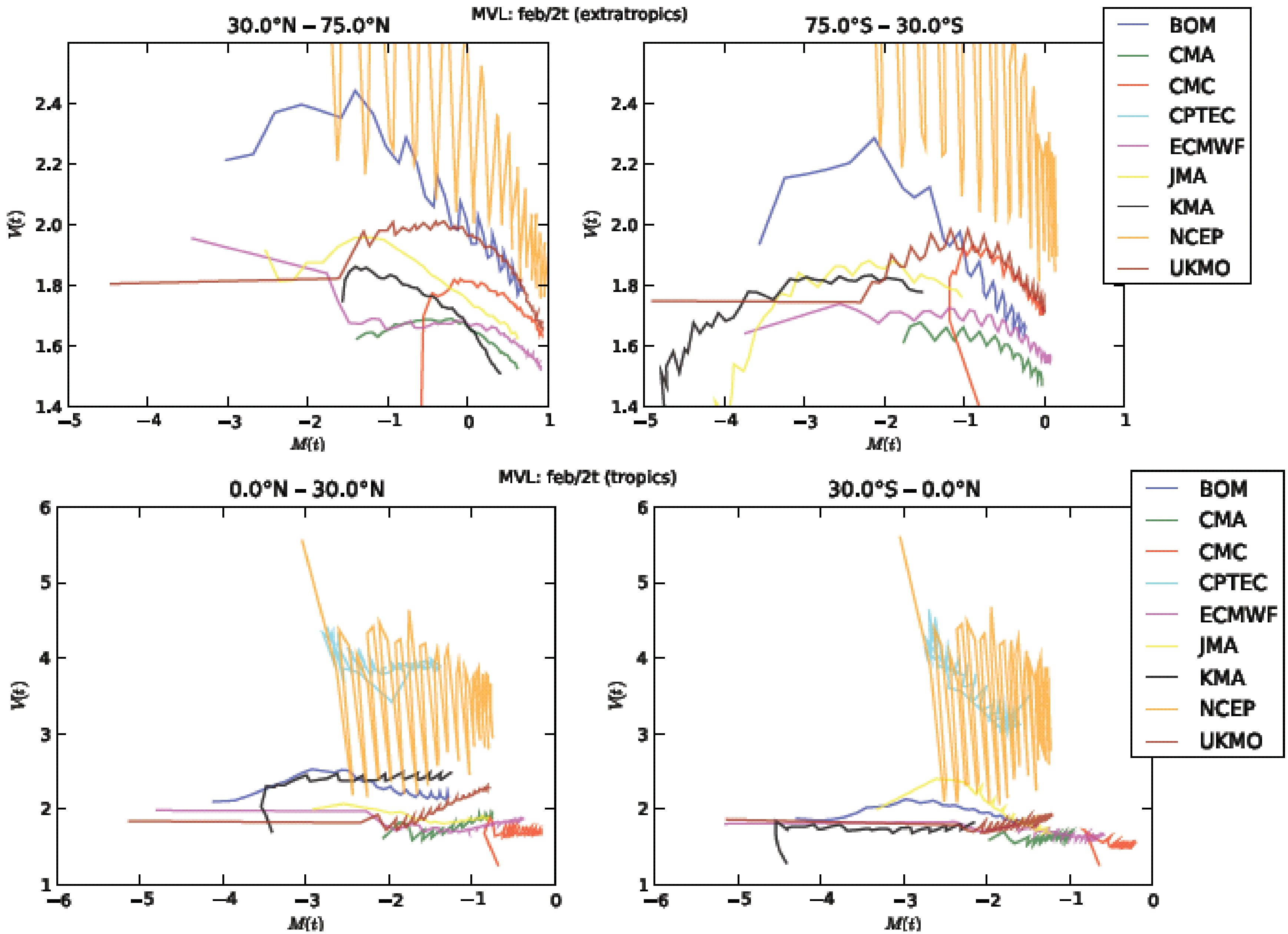
Medium-Range forecasts: TIGGE



Medium-Range forecasts: TIGGE



Medium-Range forecasts: TIGGE



Summary and conclusions

- Perturbations have an exponential growth, get spatially correlated and localized.
- A logarithmic transformation allows us to link perturbations and rough interfaces growth, providing scaling properties already obtained in this field.
- Different initial perturbation techniques have been compared using the MVL diagram.
- This diagram is shown to be an useful tool to analyze the spatiotemporal growth of perturbations, distinguising the linear and non linear regime and the correlation and amplitude growth.
- When comparing different initial perturbation techniques applied to the ECMWF EPS, SV and BV have similar amplitude after 2 days in the North Hemisphere, and similar spatial correlation after 4-5 days.
- Perturbations in the Tropics grow more slowly and their spatial correlation seems not to be saturated after 10 days for any of the techniques.

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Any questions?